

Essays in Philosophy

Volume 9

Issue 1 *Philosophy of Disability*

Article 12

1-2008

Review of “Mathematics in Kant’s Critical Philosophy: Reflections on Mathematical Practice”

Malek K. Khazaee

California State University, Long Beach

Follow this and additional works at: <http://commons.pacificu.edu/eip>



Part of the [Philosophy Commons](http://commons.pacificu.edu/eip/)

Recommended Citation

Khazaee, Malek K. (2008) "Review of “Mathematics in Kant’s Critical Philosophy: Reflections on Mathematical Practice,”” *Essays in Philosophy*: Vol. 9: Iss. 1, Article 12.

Essays in Philosophy is a biannual journal published by Pacific University Library | ISSN 1526-0569 | <http://commons.pacificu.edu/eip/>

Essays in Philosophy

A Biannual Journal

Vol. 9, No. 1, January 2008

Book Review

Mathematics in Kant's Critical Philosophy: Reflections on Mathematical Practice, Lisa A. Shabel, New York and London: Routledge, 2003. 178 pp., Preface pp. 2, Introduction, notes, bibliography, index, figures. ISBN 0-415-93955-0, hbk \$75.00

Guided initially by distinguished members of a dissertation committee (including Paul Guyer [1998 co-translator of *CPR*] and supervisor Gary Hatfield [1997 translator of the *Prolegomena*]), and edited by the late Robert Nozick in the publisher's series of "Outstanding Dissertations," this work consists of three parts with no chapters, but each part is instead divided into several small sections, and each section is titled according to the subject of discussion. As such, the lack of chapters poses no divisional problem for this book. The problem, however, which is of major concern to this reviewer, is one of distributive unevenness to the degree that it betrays the topic and the title of this work altogether. What strikes the eye at the very outset is that while the first part is completely devoted to Euclid's *Elements* (9-39) and the second purportedly to Wolff's *Elementa* (41-90), it is only the last and shortest part (91-133) that finally comes to discuss the initially intended subject-matter of the project: *CPR* A713/B741-A738/B766. This problem, which is noticeable even at the table of contents, has in fact been addressed in the Preface where the author admits that this work "began as a dissertation on Kant's philosophy of mathematics, but evolved into a more comprehensive study of the history and philosophy of early modern mathematics" (xi). I take this candid admission to mean that, from the onset, this work had gotten completely out of hand! Furthermore, nobody, and certainly not the book itself, would consider Euclid (who occupies nearly one-third of this volume) as an "early modern" mathematician, any more than Wolff would be confused as a Kantian philosopher. Given the initial intent of *Mathematics in Kant's Critical Philosophy*, I am compelled to promptly report that its grossly off-balanced distribution is demonstrative of a catastrophic failure.

While this incoherence is to be blamed on the dissertation's lack of a thesis (as incredible as it is), the contents of this work are unfortunately no more impressive. Part 1 is wasted on Euclid's centuries-known plane geometry. It starts with definitions (of point, line, surface, angle, figure, circle, etc.), and subsequently moves on to defining the Euclidian postulates and axioms. However, because of Euclid's lack of rigor, particularly from the standpoint of axiomatic systems of geometry, the author attempts to defend him by a sort of historicist sympathy. Granted that in Euclid's time there was not as much rigor as there has been since the early modern period, it may be unfair to measure and test his proofs by the subsequent advent of tools and systems. Nonetheless, had the subject of the entire book been solely the Euclidian geometry, it would have been a worthy thesis to inquire and to explore the degree to which this lack of rigor obstructed the knowledge of the structure of the physical world for more than two millennia. In any case, the

author's contextualist stance precludes her getting involved in any critical direction, and she therefore retreats to more definitions—this time of an arithmetic variety, like unit, number, etc. (Because mathematics must start with basic definitions, it does not follow that any discussion of it must start with basic definitions, much less being consumed by them.) Subsequently, Shabel concludes this section by contending that Euclidian geometry “does not constitute a formal theory awaiting interpretation; it is rather a set of propositions demonstrated directly on . . . actually constructed geometric objects” (38). Of course, Shabel, like any other writer, has the right to choose her method of approach. It is also understandable that the issue at hand is what Kant presumably could have learned from Euclid, not what the Euclidean method lacked or could have achieved. The problem, though, is that in the end the point is not made as to why we had to go through all those known definitions, and how they relate to Kant. This problem becomes all the more significant when it is realized that there is not a single reference to Euclid in the entire *CPR*.

Part 2 is plagued with the same problem as the book as a whole. For, instead of discussing its topic (Wolff), it goes off discussing virtually anybody except Wolff for the first thirty-two pages (41-73) of the forty-nine total (41-90). Among the frustrating notables and annoying critics is Euclid (again and again), his translator Dechales, as well as Viéte, Descartes (on and off, more on than off), Tacquet, Williamson, Lamy, Barrow, Legendre, Segner, Bos, and MacLaurin—with the complete absence of Leibniz. (Leibniz had a lasting influence on Wolff [especially through their extensive correspondence on mathematical and theological issues from 1704 until Leibniz's death in 1716]; Wolff's college textbooks on mathematics, as used by Kant at Königsberg, brought Leibniz closer to Kant than would otherwise have been conceivable. It was also due to Leibniz's recommendation in 1711 that Wolff was elected to the Berlin Academy.) The long list of mathematicians and critics in such a relatively short space is itself a testimony to the fact that none could have possibly been discussed to any reasonable degree. It seems as though the urge to fill the huge gap between Euclid and Wolff in this small book resulted in pushing the initially intended central figure of this part into a tight corner-end. What is left for Wolff in the remaining sixteen pages, then, amounts to a scratch on the surface, while some of his important lessons are either ignored or forced into miniaturized endnote references in the next part. Hence, neglected in Part 2 are references to some of his important and un-translated books, like the monumental *Elementa Matheseos Universae* (5 vol., 1713-1742), *Auszug aus den Anfangs-Grunden aller Mathematischen Wissenschaften zu Bequemerem Gebrauche der Anfanger*, *Philosophia Rationalis Sive Logica*, *Anfangs-Gründe aller Mathematischen Wissenschaften*, *Kurtzer Unterricht von den Vornehmsten Mathematischen Schriften*, and his painstaking *Mathematisches Lexikon*, among others.

Similar problems exist in the last and shortest part: “Kant: Mathematics in the *Critique of Pure Reason*” (91-133). Hence, in substance, this dissertation/book is only forty-two pages long, and what is presented in defense of Kant is incomplete and unconvincing. Blameworthy is the poor choice of topic, or the neglect of the chosen topic. In any case, it must be remembered that Kant only *taught* mathematics. He did not write anything in this field. It must also be understood that he never intended to develop a comprehensive philosophy of mathematics, and all that he stated about it amounts to some scattered remarks and a small controversial section in *CPR* (“The Discipline of Pure Reason in Its Dogmatic Employment,” A713/B741-A738-B766). This section was in fact a part of his larger scheme to cut the hands of the empiricists out of a very significant area of philosophy by introducing some issues that are incidentally not philosophical, but they are about mathematics. To consider the “Discipline” merely as a part of this much larger anti-empiricist

project may lead to the question of whether an application or continuation of Wolff's idea of hypothetical stipulation, or his rules of symbolic manipulation, or his elementary conceptions, theorems, corollaries, and scholia are manifested in and employed by Kant.

Shabel insists that Wolff's distinction between the "mechanical" demonstration and its "mathematical" counterpart has inspired the distinction between "empirical" and "pure" intuitions in *CPR*. This may have been the case. Or maybe not. The reason for caution is twofold:

First, in the entire voluminous *CPR*, Wolff's name is mentioned only twice: in the Preface to Second Edition and in the "Aesthetic" (none in the "Discipline"). In the former, the reference to Wolff is very general: being praised as "the greatest of all dogmatic philosophers," and blamed for failing to prepare "the ground beforehand by a critique of . . . pure reason itself" (Bxxxvi). In the latter, he is named, along with Leibniz, as having taken "a completely wrong direction" by "treating the difference between the sensible and the intelligible as merely logical" (A44).

Second and more important than the "Discipline's" lack of reference to Wolff is its contrasting approach to his aforementioned distinction. For, Wolff's "mechanical demonstration" is provided as the ground for constructing "mathematical demonstration," while Kant's "empirical intuition" comes only after the "pure intuition" has already been formed in the imagination. With regard to Wolff's two separate ways of proving Euclid's thirty-second proposition (*Elements*, Book I), Shabel correctly states that "'the mechanical demonstrations' are especially useful insofar as they lead to the mathematical demonstration of the same proposition" (99). But she overlooks the order of this Wolffian procedure when speaking of the relation of "pure" to "empirical intuition" in Kant. For, as a matter of fact, this Wolffian approach, which is intended for the practical employment of instruments (open compass, rulers, etc.), is clearly *opposite* to the Kantian procedure in which a triangle is constructed "either by imagination alone, in pure intuition, or *in accordance therewith* [stress added] also on paper, in empirical intuition" (A713/B741). Hence, because of Kant's lack of reference to Wolff on one hand, and his opposite approach on the other, it is impossible to be certain that Kant got the above distinction from Wolff. He may have done so (by deliberately reversing the Wolffian order), or he may not (by forgetting the original source and conceiving it reversely in the heat of scoring a significant point against the empiricists).

Kant seems most interested in arguing that even though a drawn "empirical intuition" is individual, it nevertheless expresses the concept universally insofar as it borrows its pattern from a non-experiential source (A713/B741-A714/B742). The way to reach this point starts with the distinction he famously makes between the foundations of mathematical and philosophical knowledge, whereby the attainment of the former, he claims, is the best example of the *construction* of concepts in mere imagination by the dogmatic employment of pure reason (A713/B741). This claim enables him to demarcate philosophical cognition which, in his view, is acquired "*by reason from concepts*" from mathematical cognition which, he insists, is "gained by reason from the *construction* of concepts." He immediately adds, "To *construct* a concept means to exhibit *a priori* the intuition which corresponds to the concept. For the construction of a concept we therefore need a *non-empirical* intuition" (ibid.). Hence, the distinction between pure (universal) intuition and empirical (particular) intuition as well as their similitude in "both" being "completely *a priori*" (ibid.). To avoid objections (or "confusion" in Kant's mind) about the term "empirical intuition," he further argues that, in spite of its being drawn on paper as an individual image, it has *borrowed* its form from the non-experiential realm (A714/B742).

While this discussion could serve as an appropriate *starting* point for a work on Kant's view of mathematics, it is the point at which Shabel *finally* arrives in hoping to solve the inherent tension of Kant's synthetic apriority of mathematical cognition. She therefore tries to land on an impossible ground in the few remaining pages of the book. The difficulty of defending this historically disputed notion—that mathematical cognition is synthetic *a priori*—is indeed so enormous that no clear-cut resolution has so far come to bear. In fact, soon after the publication of *CPR*, objections were raised against Kant's assignment of "pure intuition" as the basis of mathematical cognition. (One such objection was brought by Eberhard whose view is quoted by Shabel from a secondary, pro-Kantian source, and subsequently underscored by her in an uneven, biased manner [106-9]. This is a pattern in treating favorites against their adversaries.) In addition, several significant challenges have occurred afterwards. For example, Kant's extension of synthetic apriority from geometry to arithmetic was criticized by Frege to the degree that one can still say that the proposition " $7+5=12$ " is analytic. Likewise, the supposed move from geometry's "ostensive construction" of the concepts to the algebraic "symbolic construction" (the latter branch mentioned only in A717/B745 and A734/B762) has been attacked by many critics as ambiguous and unconvincing. It has also been objected that the idea of "symbolic construction" has no established connection with the synthetic apriority of the "Aesthetic's" Space and Time (B41-B73). In addition, one should not forget Quine's "Two Dogmas of Empiricism," which vanished the age-old distinction between analytic and synthetic judgments.

Of course, these reminders are merely a few highlights, not intended to amount to a critique of Kant's critical philosophy. The point, however, is that defending Kant in a few pages is undefendable. I do not know what to make of Shabel's last words, that Kant's "reflections on eighteenth century mathematical practice continue to inform and influence our contemporary philosophical and mathematical investigations." Is this really happening today on this planet? Blameworthy for this reactionary approach is, perhaps, the committee whose influence might have deprived the enthusiastic candidate from developing a more critical and liberated stance toward the old Prussian master.

Malek K. Khazaei
California State University, Long Beach